Microwave Circuits and Antenna Design

Amplifier Design
&
S-Parameters

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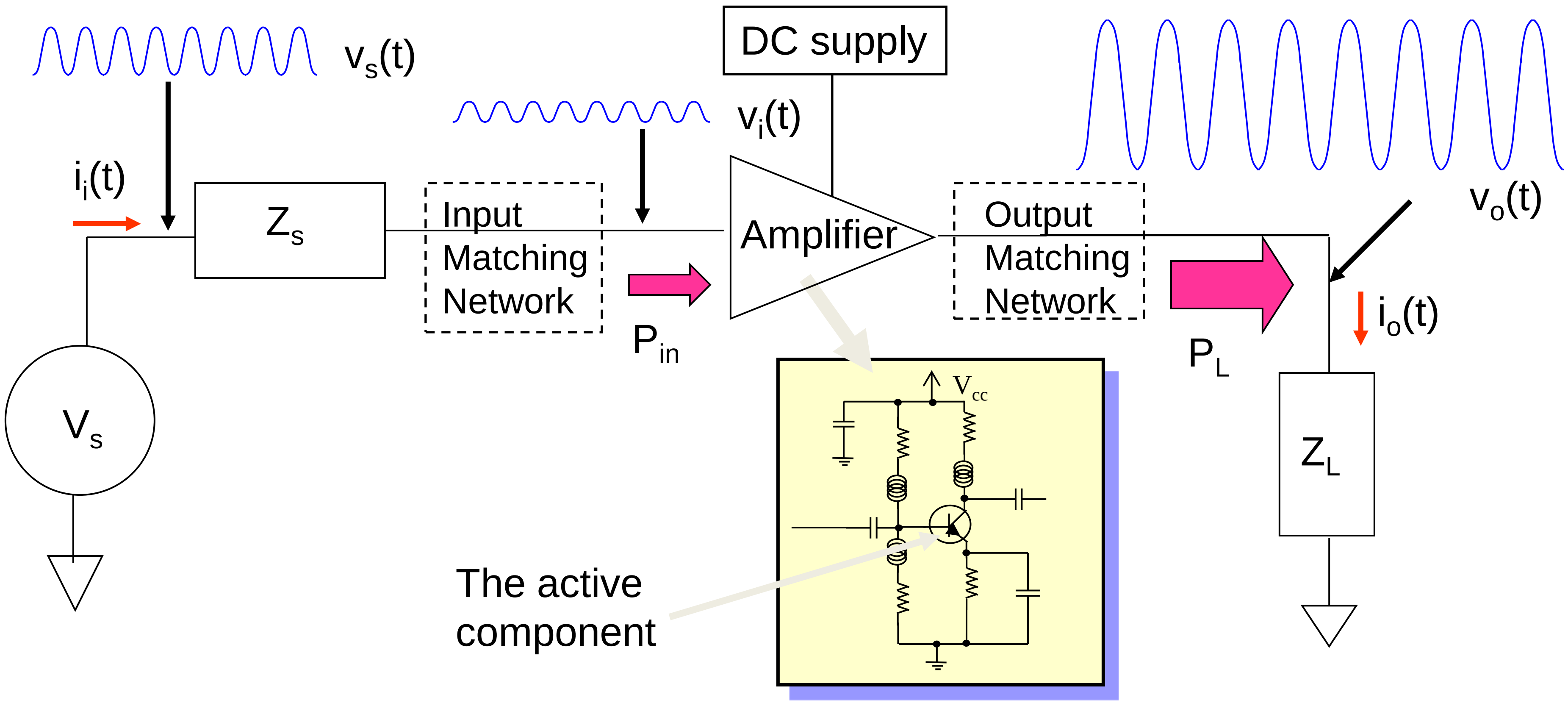
General Amplifier Block Diagram



Input and output voltage relation of the amplifier can be modeled simply as:

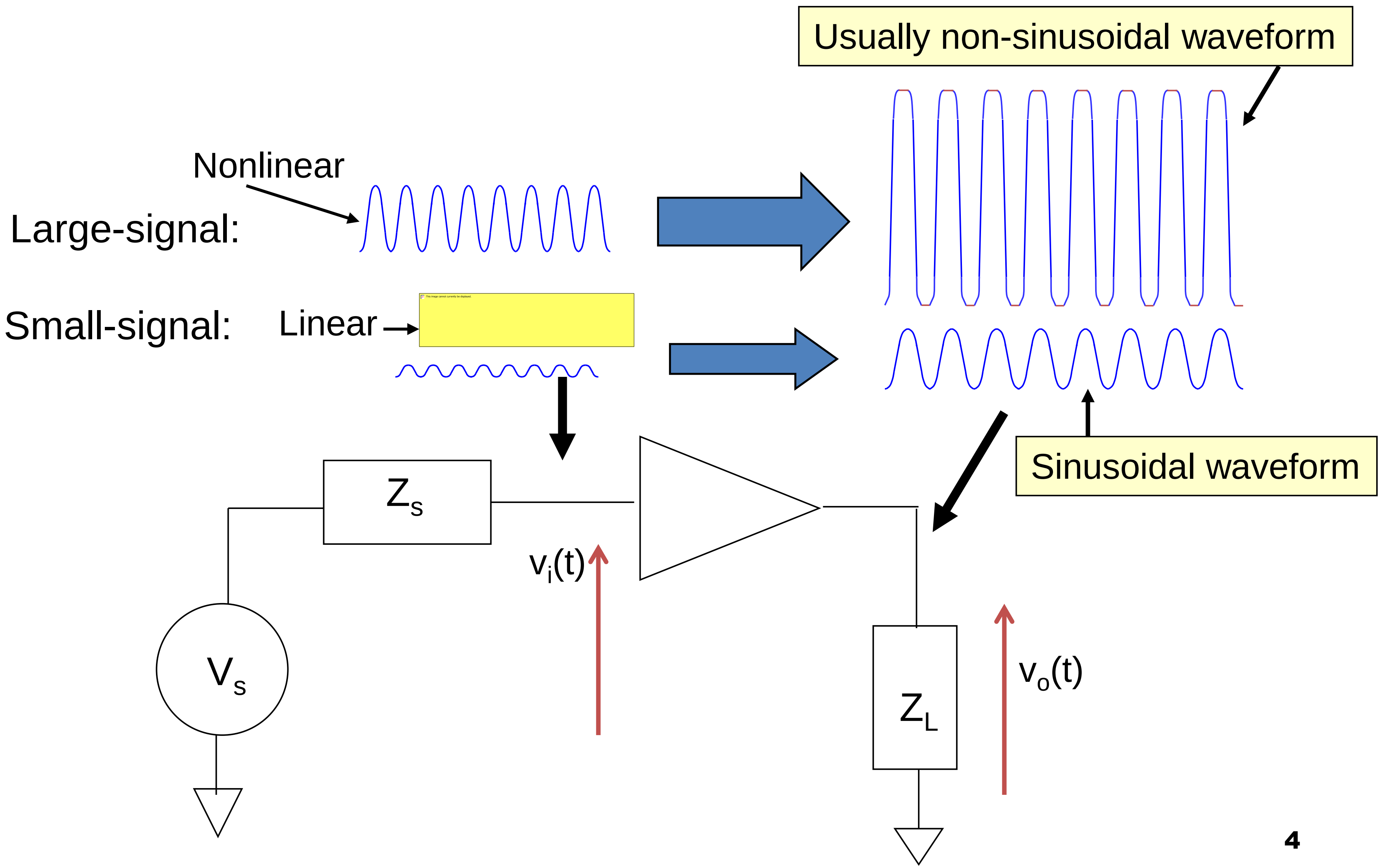
$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) + H.O.T.$$

Amplifier Classification

- Amplifier can be categorized in 2 manners.
- According to signal level:
  - Small-signal Amplifier.
  - Power/Large-signal Amplifier.
- According to D.C. biasing scheme of the active component:
  - Class A.
  - Class B.
  - Class AB.
  - Class C.

There are also other classes, such as Class D (D stands for digital), Class E and Class F. These all uses the transistor/FET as a switch.

Small-Signal Versus Large-Signal Operation



Small-Signal Amplifier (SSA)

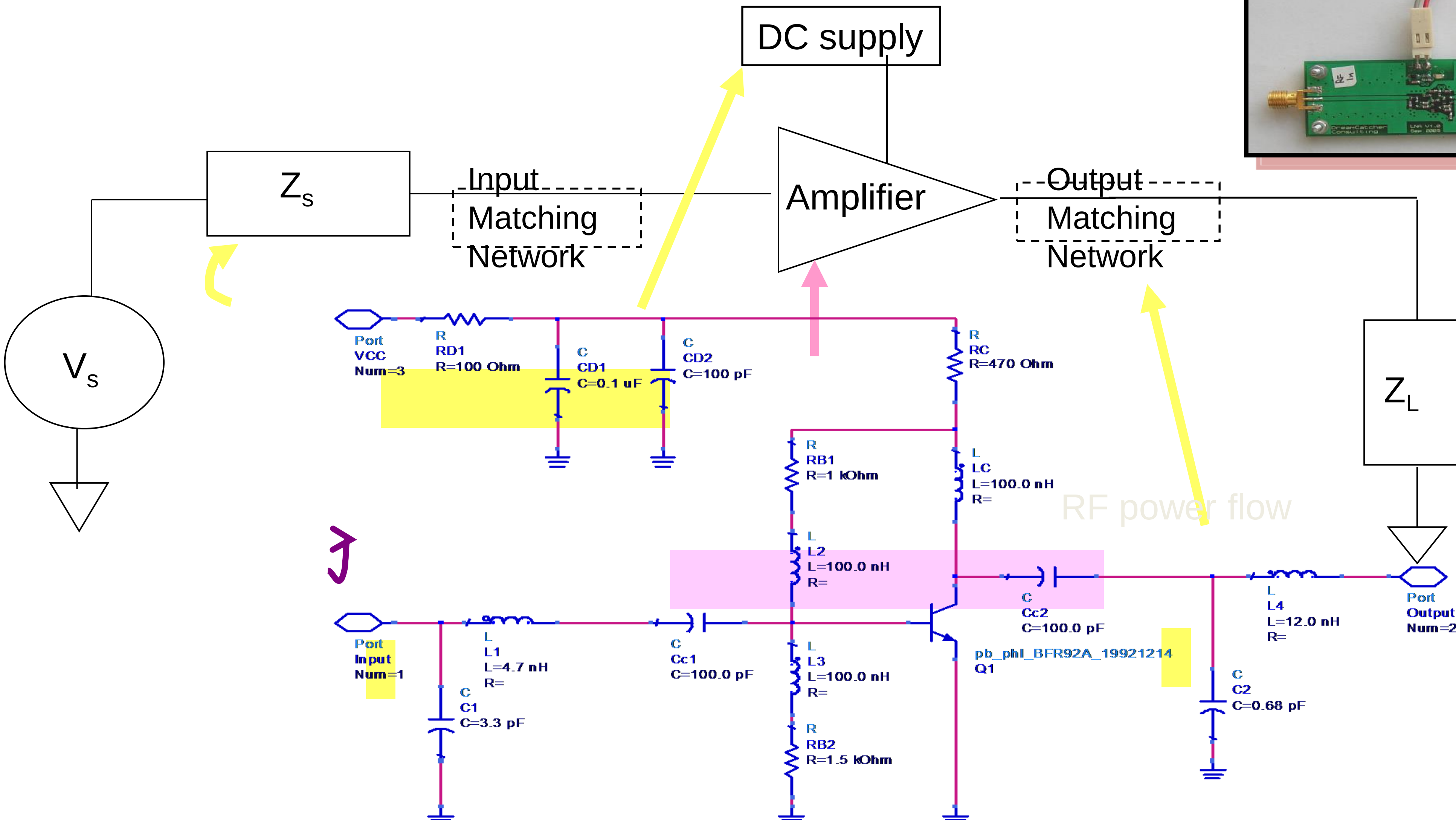
- All amplifiers are inherently nonlinear.
- However when the input signal is small, the input and output relationship of the amplifier is approximately linear.

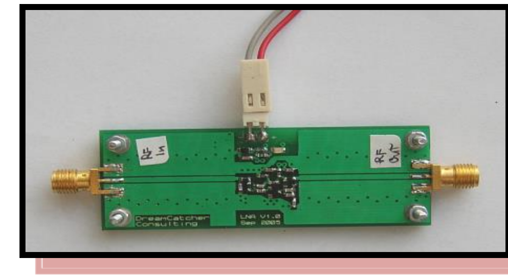
Linear relation

When $v_i(t) \rightarrow 0$ (< 2.6mV) → $$v_o(t) \cong a_1 v_i(t) \quad \textbf{(1.1)}$$

- This linear relationship applies also to current and power.
- An amplifier that fulfills these conditions: (1) small-signal operation (2) linear, is called Small-Signal Amplifier (SSA). SSA will be our focus.
- If a SSA amplifier contains BJT and FET, these components can be replaced by their respective small-signal model, for instance the hybrid-Pi model for BJT.

An RF Amplifier Schematic





Typical RF Amplifier Characteristics

- To determine the performance of an amplifier, the following characteristics are typically observed.
- 1. Power Gain.
- 2. Bandwidth (operating frequency range).
- 3. Noise Figure.
- 4. Phase response.
- 5. Gain compression.
- 6. Dynamic range.
- 7. Harmonic distortion.
- 8. Intermodulation distortion.
- 9. Third order intercept point (TOI).

Important to small-signal amplifier (items 1–4)

Important parameters of large-signal amplifier (Related to Linearity) (items 5–9)

Power Gain

- For amplifiers functioning at RF and microwave frequencies, usually of interest is the input and output power relation.
- The ratio of output power over input power is called the **Power Gain (G)**, usually expressed in dB.

Power Gain

$$G = 10\log_{10}\left(\frac{\text{Output Power}}{\text{Input Power}}\right)\ \text{dB} \qquad \textbf{(1.2)}$$

- There are a number of definition for power gain as we will see shortly.
- Furthermore G is a function of frequency and the input signal level.

Why Power Gain for RF and Microwave Circuits? (1)

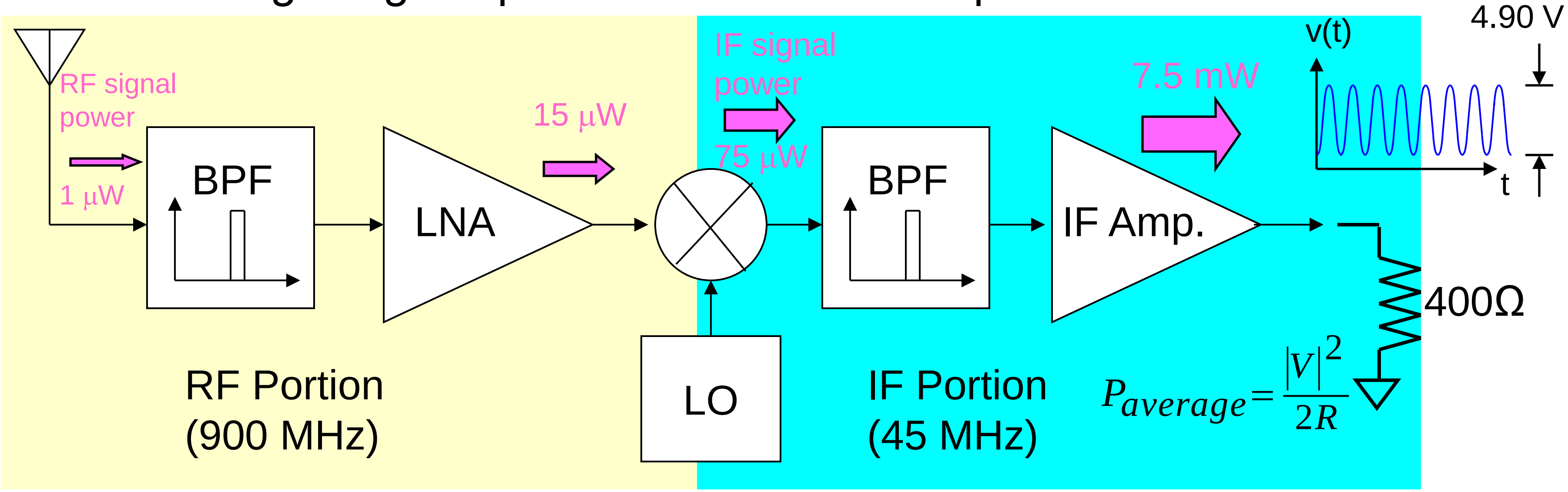
- Power gain is preferred for high frequency amplifiers as the impedance encountered is usually low (due to presence of parasitic capacitance).

Power = Voltage x Current

- For instance if the amplifier is required to drive 50Ω load the voltage across the load may be small, although the corresponding current may be large (there is current gain).
- For amplifiers functioning at lower frequency (such as IF frequency), it is the voltage gain that is of interest, since impedance encountered is usually higher (less parasitic).
- For instance if the output of IF amplifier drives the demodulator circuits, which are usually digital systems, the impedance looking into the digital system is high and large voltage can developed across it. Thus working with voltage gain is more convenient.

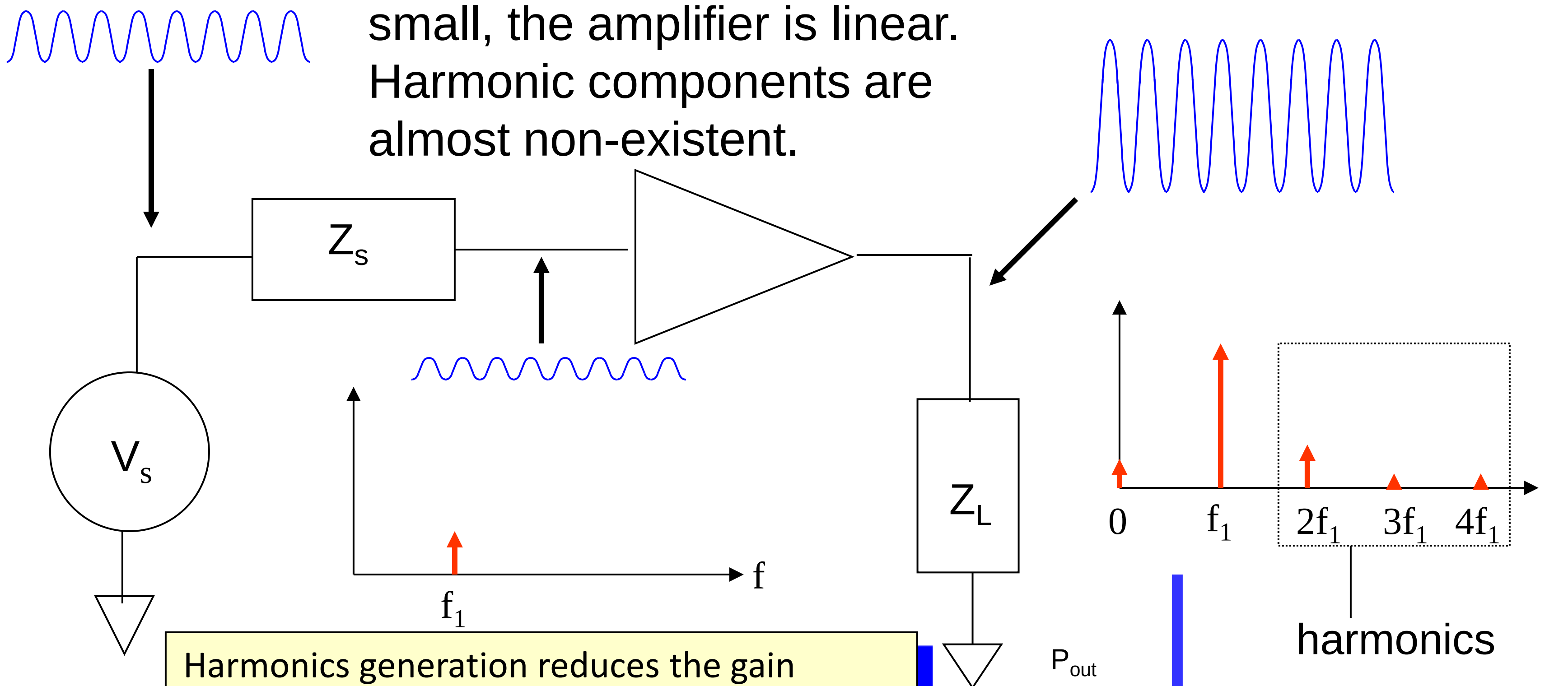
Why Power Gain for RF and Microwave Circuits? (2)

- Instead on focusing on voltage or current gain, RF engineers focus on power gain.
- By working with power gain, the RF designer is free from the constraint of system impedance. For instance in the simple receiver block diagram below, each block contribute some power gain. A large voltage signal can be obtained from the output of the final block by attaching a high impedance load to it's output.

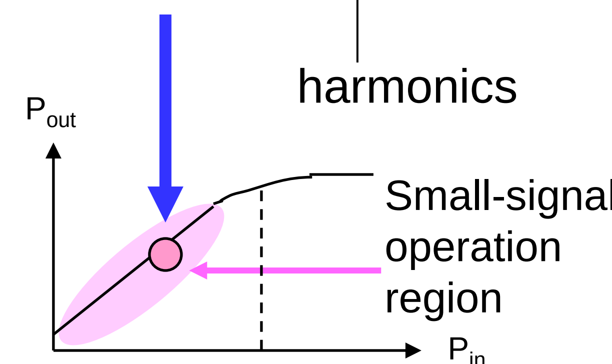


Harmonic Distortion (1)

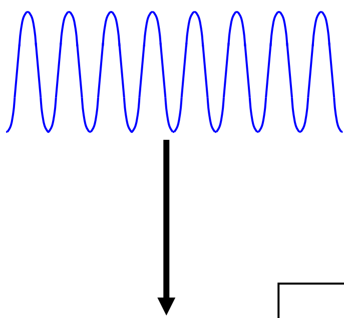
When the input driving signal is small, the amplifier is linear. Harmonic components are almost non-existent.



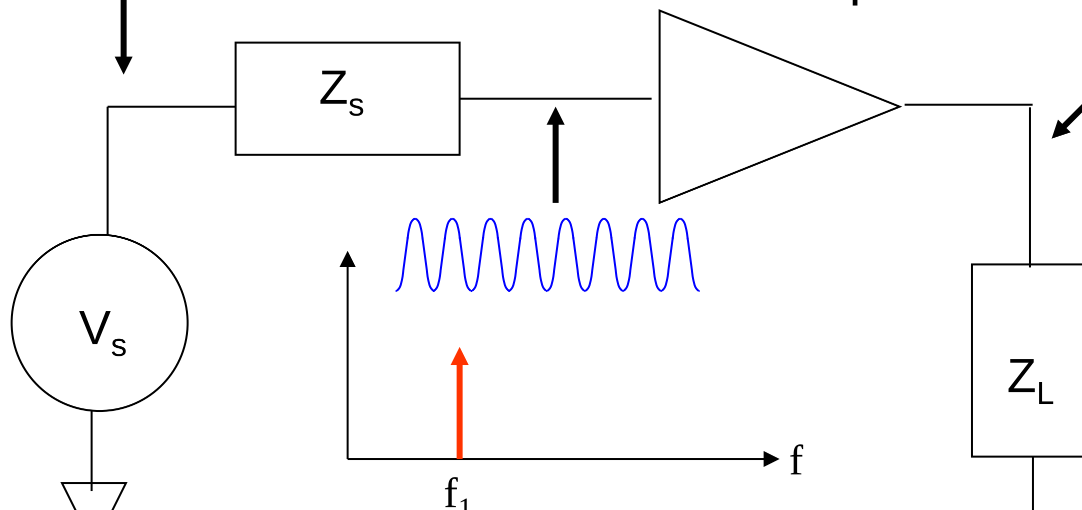
Harmonics generation reduces the gain of the amplifier, as some of the output power at the fundamental frequency is shifted to higher harmonics. This result in **gain compression** seen earlier!

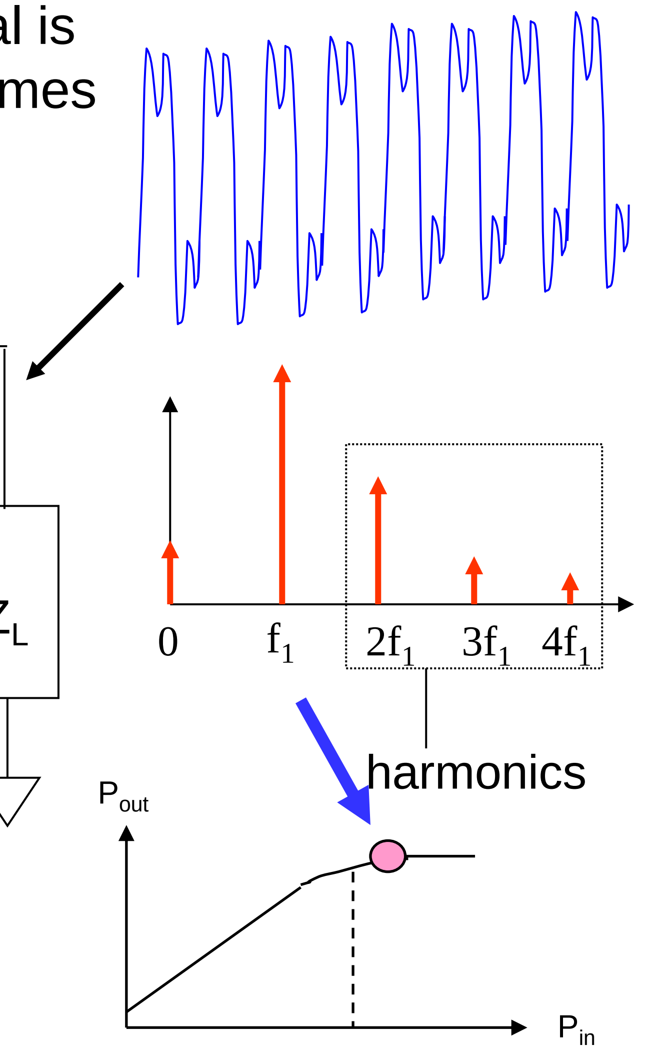


Harmonic Distortion (2)



When the input driving signal is too large, the amplifier becomes nonlinear. Harmonics are introduced at the output.





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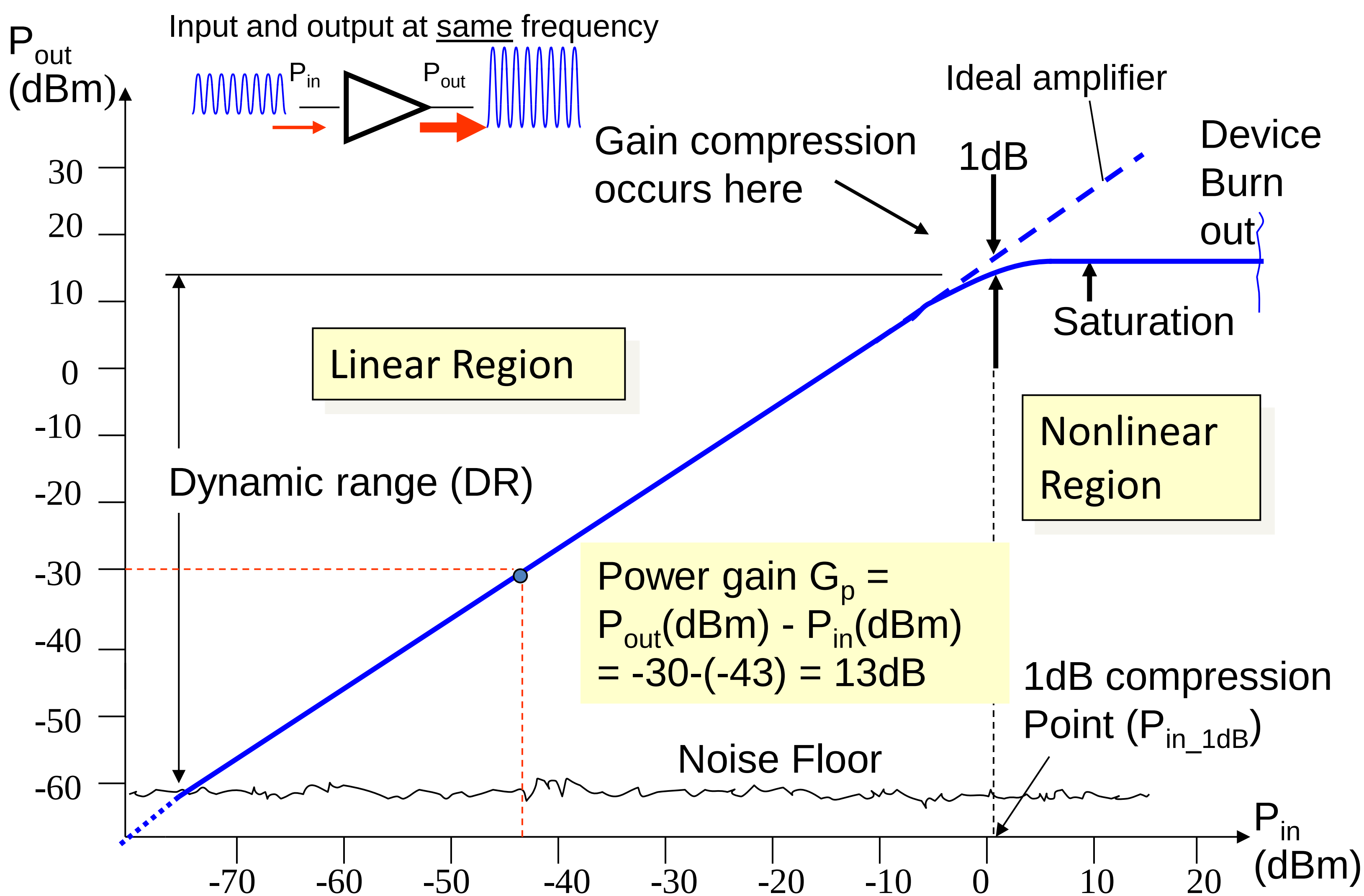
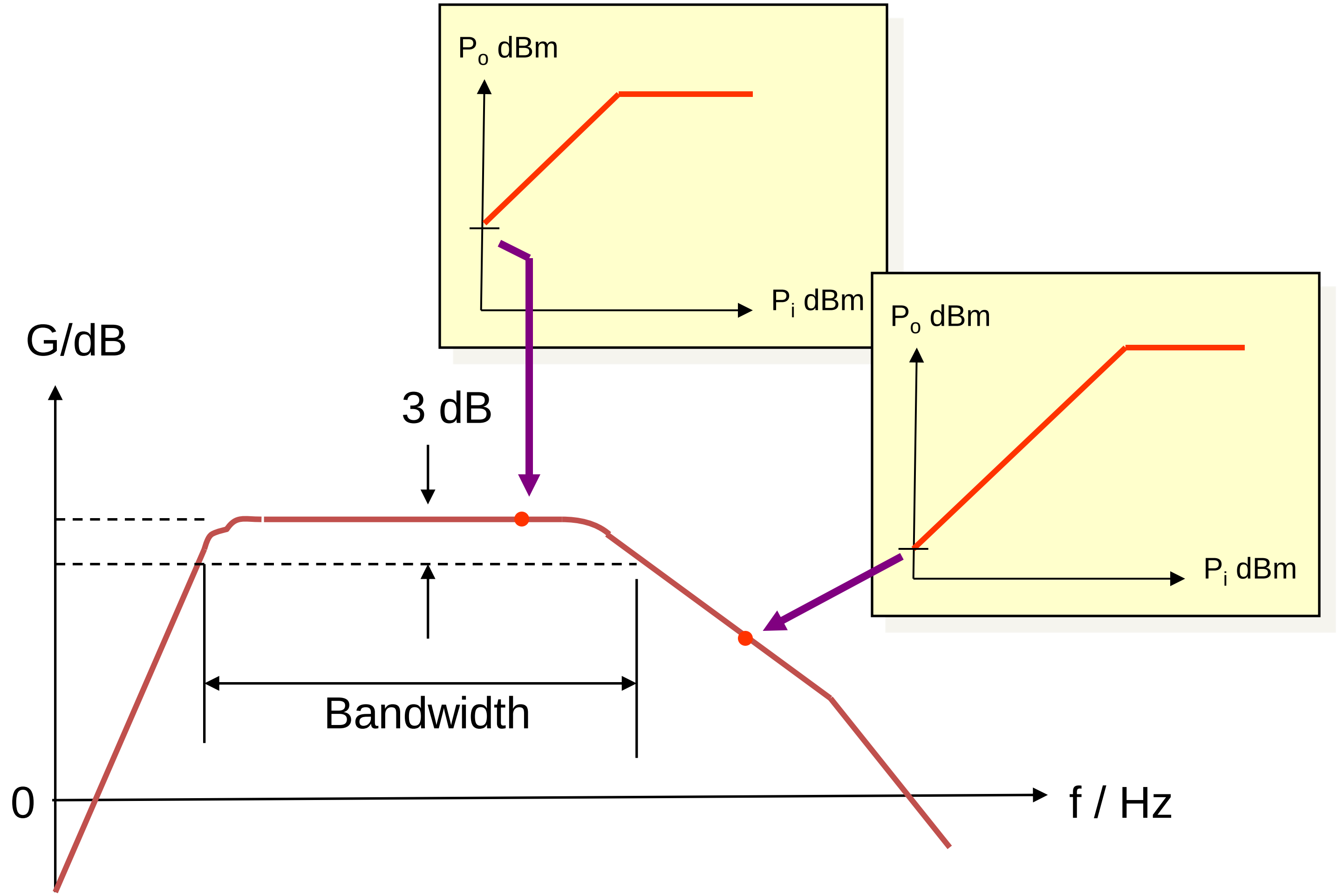
Power Gain, Dynamic Range and Gain Compression



Bandwidth

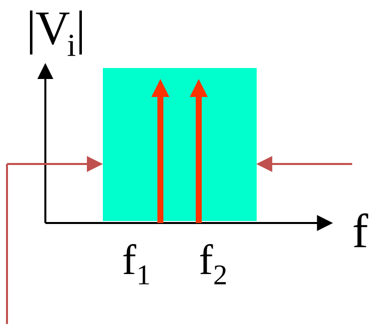
- Power gain G versus frequency for small-signal amplifier.

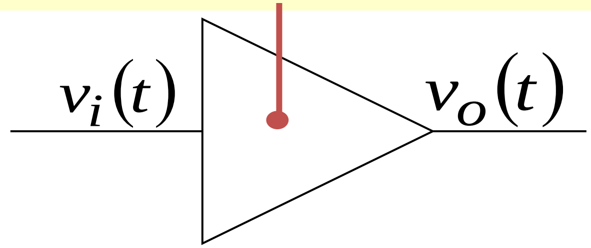


Intermodulation Distortion (IMD)

$$v_o(t) = a_1 v_i(t) + a_2 v_i^{\,2}(t) + a_3 v_i^{\,3}(t) + H.O.T.$$

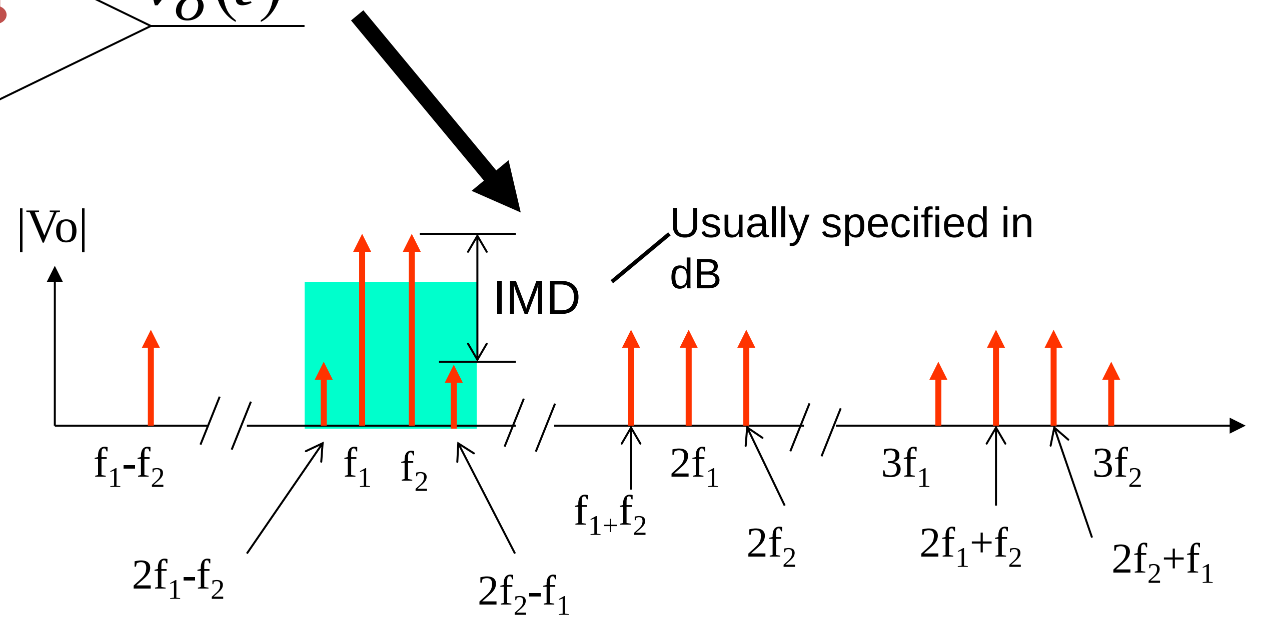
ignored





Operating bandwidth of the amplifier

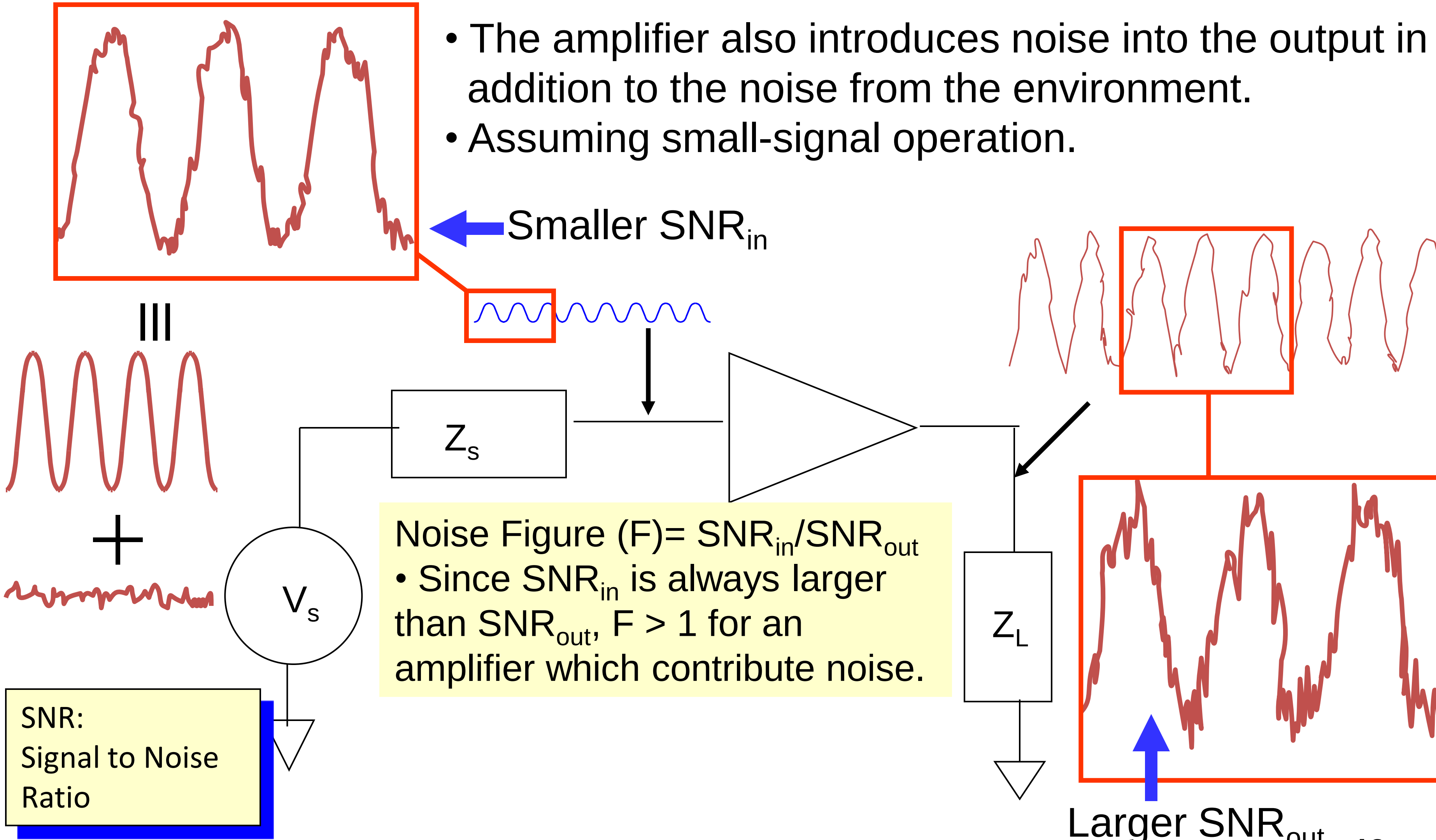
Two signals v_1, v_2 with similar amplitude, frequencies f_1 and f_2 near each other



These are unwanted components, caused by the term $\alpha_3 v_i^3(t)$, which falls in the operating bandwidth of the amplifier.

Noise Figure (F)

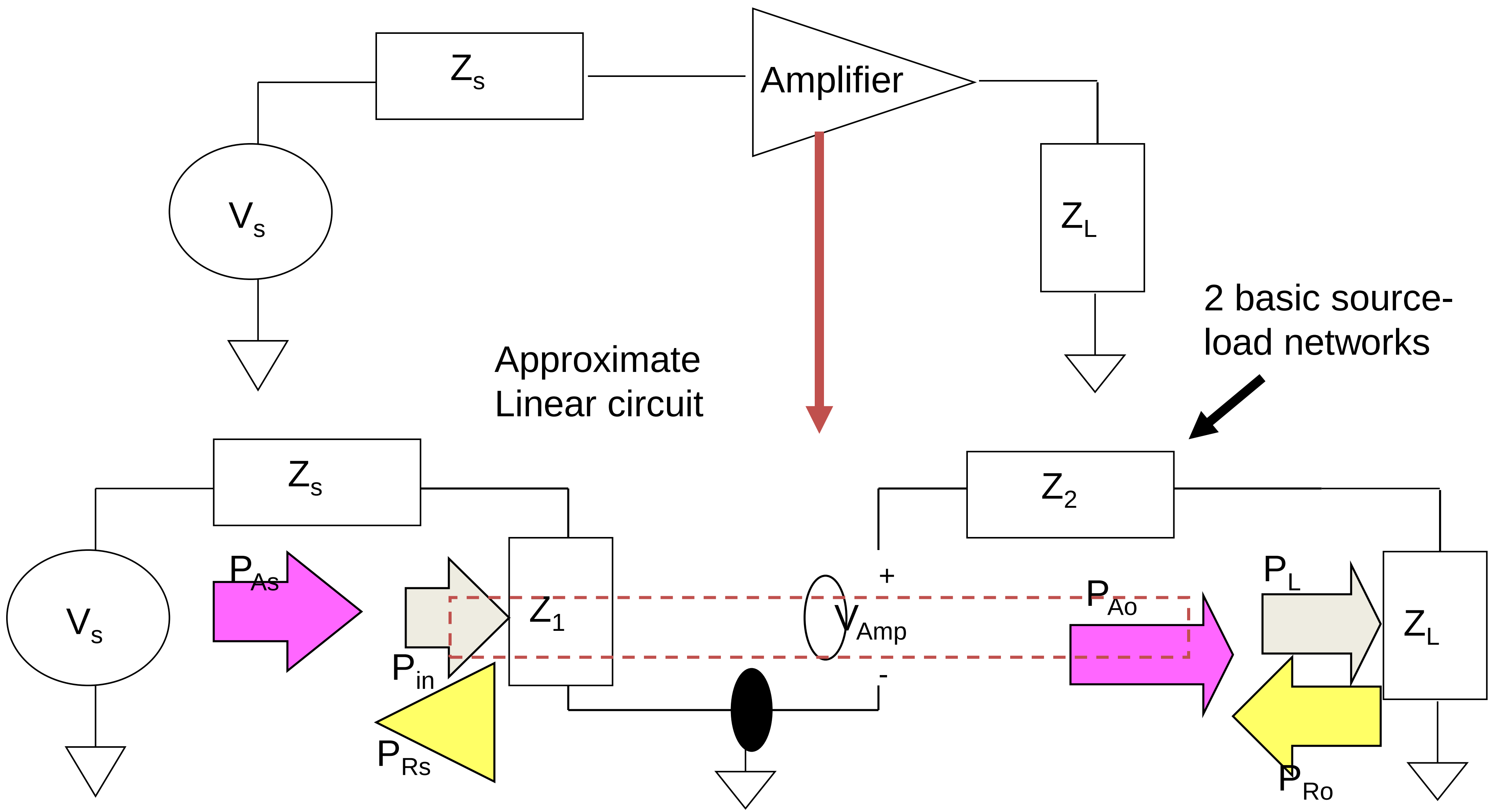
- The amplifier also introduces noise into the output in addition to the noise from the environment.
- Assuming small-signal operation.



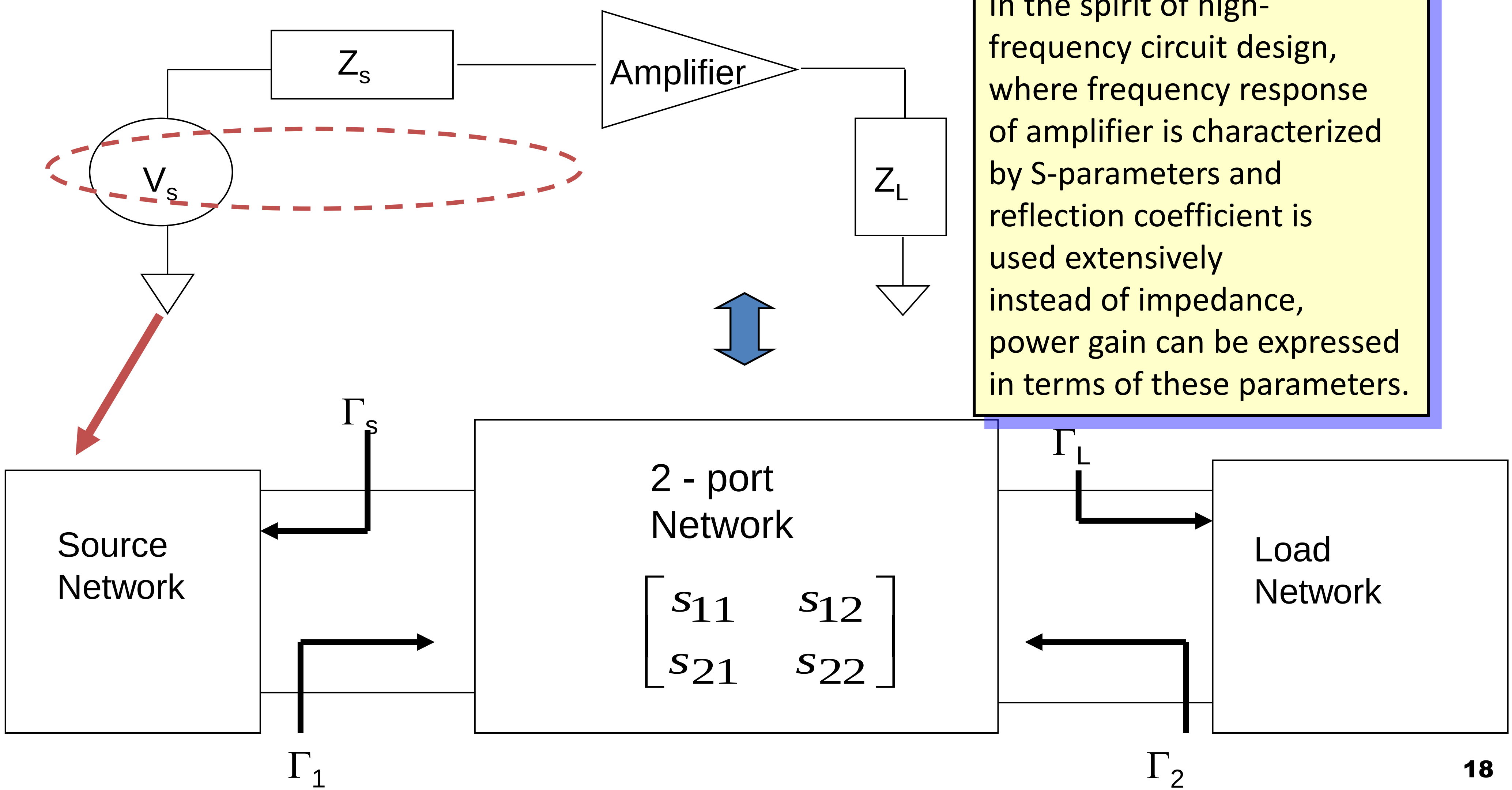
Noise Figure (F)= SNR_{in}/SNR_{out}
- Since SNR_{in} is always larger than SNR_{out}, F > 1 for an amplifier which contribute noise.

SNR:
Signal to Noise Ratio

Power Components in an Amplifier



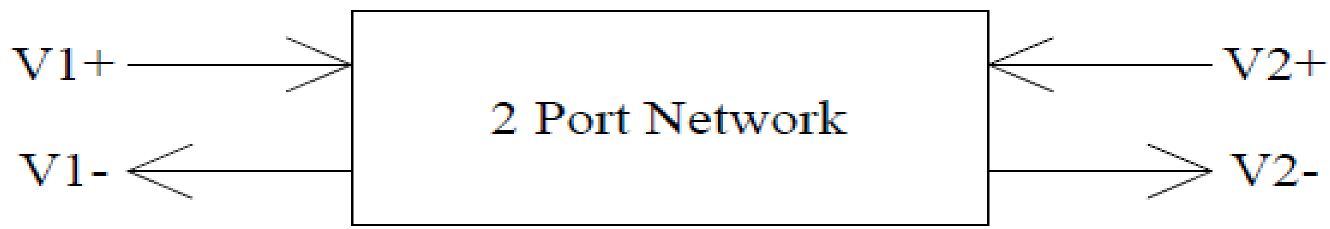
Naming Convention



In the spirit of high-frequency circuit design, where frequency response of amplifier is characterized by S-parameters and reflection coefficient is used extensively instead of impedance, power gain can be expressed in terms of these parameters.

Scattering Parameters

Scattering Parameters



S parameter definitions

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

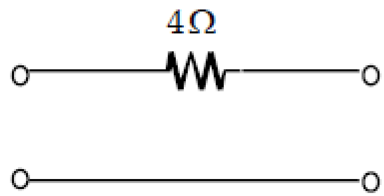
$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0}$$ i.e. reflection coefficient at port 1 with an ideal match at port 2

$$S_{22} = \left. \frac{V_2^-}{V_2^+} \right|_{V_1^+ = 0}$$ i.e. reflection coefficient at port 2 with an ideal match at port 1

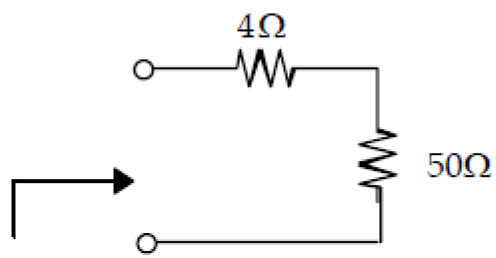
$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0}$$ i.e forward gain

$$S_{12} = \left. \frac{V_1^-}{V_2^+} \right|_{V_1^+ = 0}$$ i.e reverse gain, i.e effect of output on input.

Example:



Given $Z_0 = 50\Omega$, what is S_{11}?



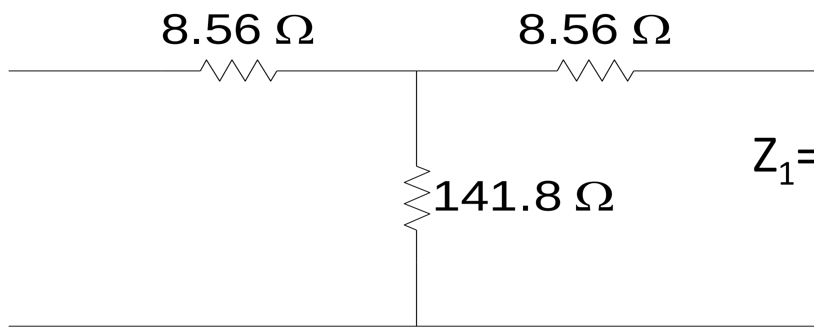
$$Z_{in}\big|_{Z_L = Z_0} = 54\Omega$$

$$S_{11} = \frac{54 - 50}{54 + 50} = \frac{4}{104}$$

Similar arguments give $S_{22} = \dfrac{4}{104}$.

Example

Below is a matched 3 dB attenuator. Find the S-parameter of the circuit.



Z_1=Z_2= 8.56 Ω and Z_3= 141.8 Ω

Solution

$$S_{11} = \left.\frac{V_{r1}}{V_{i1}}\right|_{V_{r2}=0} = \rho = \frac{Z_{in} - Z_o}{Z_{in} + Z_o}$$

By assuming the output port is terminated by Z_o = 50 Ω, then

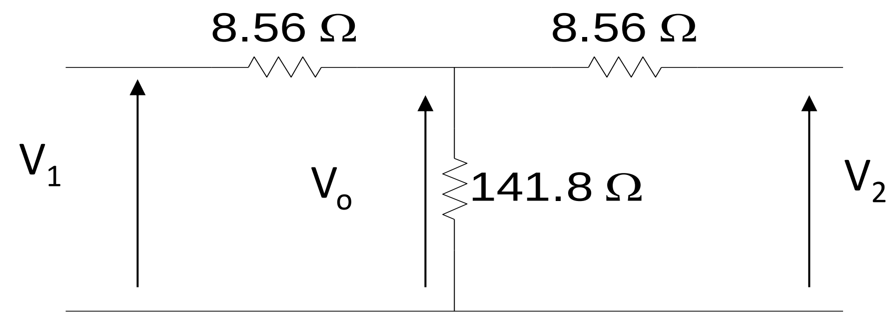
$$Z_{in} = Z_1 + \left[Z_3 // (Z_2 + Z_o)\right]$$

$$= 8.56 + \left[141.8(8.56+50)/(141.8+8.56+50)\right] = 50\ \Omega$$

$$S_{11} = \frac{50-50}{50+50} = 0$$

Because of symmetry , then S_{22}=0

$$S_{21} = \left.\frac{V_{t2}}{V_{i2}}\right|_{V_{r2}=0}$$



From the fact that $S_{11}=S_{22}=0$, we know that $V_{r1}=0$ when port 2 is matched, and that $V_{i2}=0$. Therefore $V_{i1}= V_1$ and $V_{t2}=V2$

$$V_{t2} = V_2 = V_1\left(\frac{Z_2 // Z_3}{Z_2 // Z_3 + Z_1}\right)\left(\frac{Z_o}{Z_3 + Z_o}\right) = V_o\left(\frac{Z_o}{Z_3 + Z_o}\right)$$

$$= V_1\left(\frac{41.44}{41.44+8.56}\right)\left(\frac{50}{50+8.56}\right) = 0.707V_1$$

Therefore $S_{12} = S_{21} = 0.707$

$$[S] = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

Example

A certain two-port network is measured and the following scattering matrix is obtained:

$$[S]=\begin{bmatrix} 0.1\angle 0^o & 0.8\angle 90^o \\ 0.8\angle 90^o & 0.2\angle 0^o \end{bmatrix}$$

From the data , determine whether the network is reciprocal or lossless. If a short circuit is placed on port 2, what will be the resulting return loss at port 1?

Solution

Since [S] is symmetry, the network is reciprocal. To be lossless, the S parameters must satisfy

$$\sum_{k=1}^{n} S_{ki}S_{kj}^{*} = \begin{matrix} 1 & for\, i = j \\ 0 & for\, i \neq j \end{matrix}\Bigg\}$$

For i=j

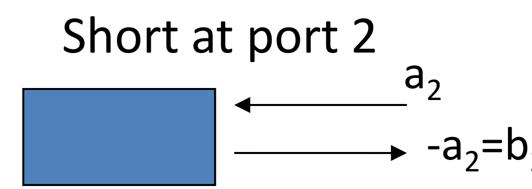
$|S_{11}|^2 + |S_{12}|^2 = (0.1)^2 + (0.8)^2 = 0.65$

Since the summation is not equal to 1, thus it is not a lossless network.

Reflected power at port 1 when port 2 is shorted can be calculated as follow and the fact that a_2= $-b_2$ for port 2 being short circuited, thus

$$b_1=S_{11}a_1 + S_{12}a_2 = S_{11}a_1 - S_{12}b_2 \quad (1)$$

$$b_2=S_{21}a_1 + S_{22}a_2 = S_{21}a_1 - S_{22}b_2 \quad (2)$$



From (2) we have

$$b_2 = \frac{S_{21}}{1+S_{22}} a_1 \quad (3)$$

Dividing (1) by a_1 and substitute the result in (3) ,we have

$$\rho = \frac{b_1}{a_1} = S_{11} - S_{12}\frac{b_2}{a_1} = S_{11} - \frac{S_{12}S_{21}}{1+S_{22}} = 0.1 - \frac{(j0.8)(j0.8)}{1+0.2} = 0.633$$

Return loss $-20\log\rho = -20\log(0.633) = 3.97\,dB$

EXAMPLE

A two-port net work is measured and the following scattering matrix is obtained:

$$[S]=\begin{bmatrix} 0.15\angle 0^o & 0.85\angle -45^o \\ 0.85\angle 45^o & 0.2\angle 0^o \end{bmatrix}$$

a) determine whether the network is reciprocal or lossless.

b) If port two is terminated with a matched load, what will be the return loss at port 1?

c) If a short-circuit is placed on port 2, what will be the resulting return loss at port 1?

Sol

a) Since [S] is not symmetry, the net work is reciprocal.

To be lossless, the [S] parameters must satisfy (4.53). Since

$$|S_{11}|^2+|S_{22}|^2=0.15^2+0.85^2=0.745\neq 1$$

Thus, the network is not lossless.

b) When port 2 is terminated with matched load, the reflection coefficient at port 1 is $\Gamma = S_{11} = 0.15$, Thus,

$$RL = -20\log|\Gamma| = -20\log(0.15) = 16.5dB$$

- When port 2 is terminated with a short circuit, we have Thus,

$$V_2^+ = -V_2^-$$

- The second equation gives

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+ = S_{11}V_1^+ - S_{12}V_2^-,$$

- Substituting in

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ - S_{22}V_2^-.$$

$$V_2^- = \frac{S_{21}}{1 + S_{22}} V_1^+.$$

$$\Gamma = \frac{V_1^-}{V_1^+} = S_{11} - S_{12}\frac{V_2^-}{V_1^+} = S_{11} - \frac{S_{12}S_{21}}{1 + S_{22}}$$

$$= 0.15 - \frac{(0.85\angle -45^o)(0.85\angle 45^o)}{1 + 0.2} = -0.452$$

- So the return loss is

$$RL = -20\log|\Gamma| = -20\log(0.452) = 6.9dB$$

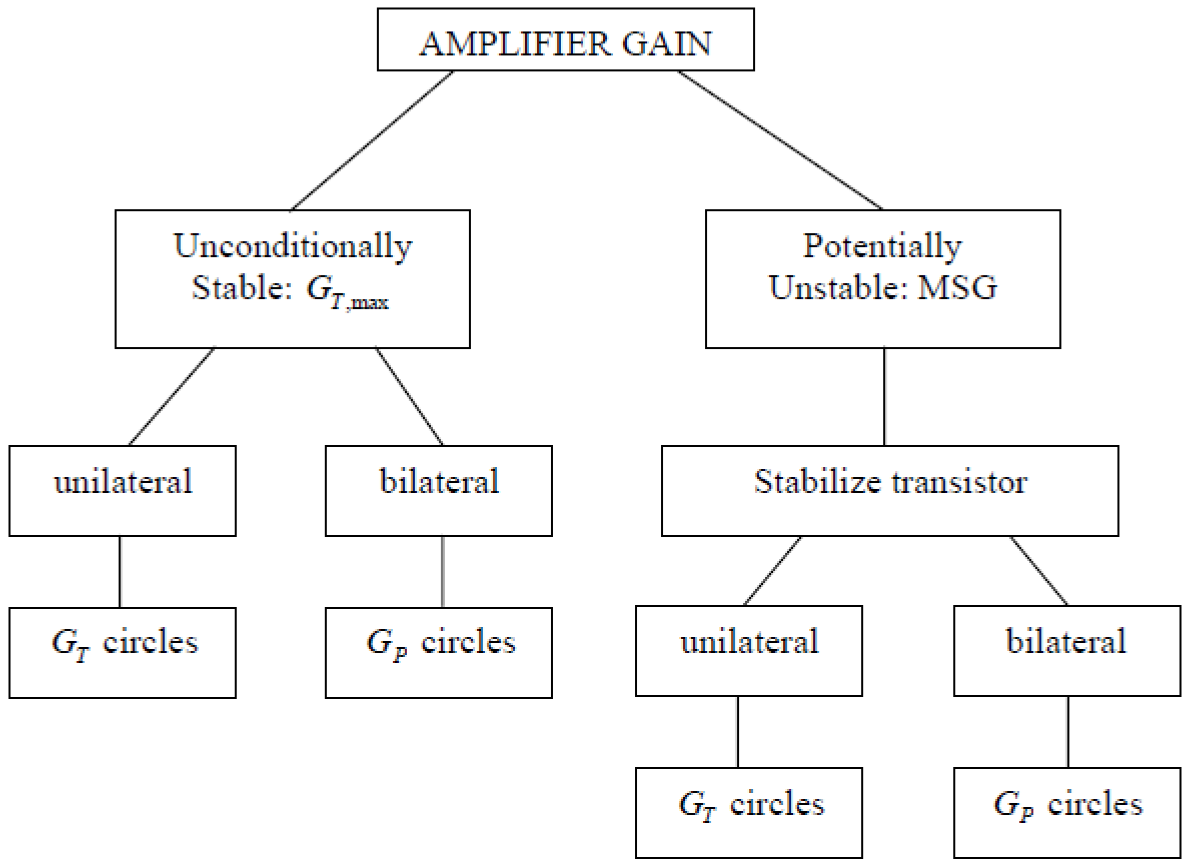
NOTE

- The reflection coefficient looking into port n is not equal to S_{nn} unless all other ports are matched.
- Similarly, the transmission coefficient from port *m* to port *n* is not equal to S_{nm}, unless all other ports are matched.
- The parameters of a network are properties only of the network itself (assuming the network is linear), and are defined under the condition that all ports are matched.

Microwave Amplifies Design

Amplifier Gain

At a given frequency, the maximum gain that an amplifier can deliver is limited by either its $G_{max} = G_{T,max}$ or by stability G_{MSG}



Microwave Amplifier Design

Two-port power gains

- power gains G, G_T, G_A

Stability

- input and output stability circles, stability criterion

Single-stage transistor amplifier design

- conjugate match, constant gain circle, noise parameters, constant noise figure circle, LNA (low noise amplifier)

Broadband transistor amplifier design

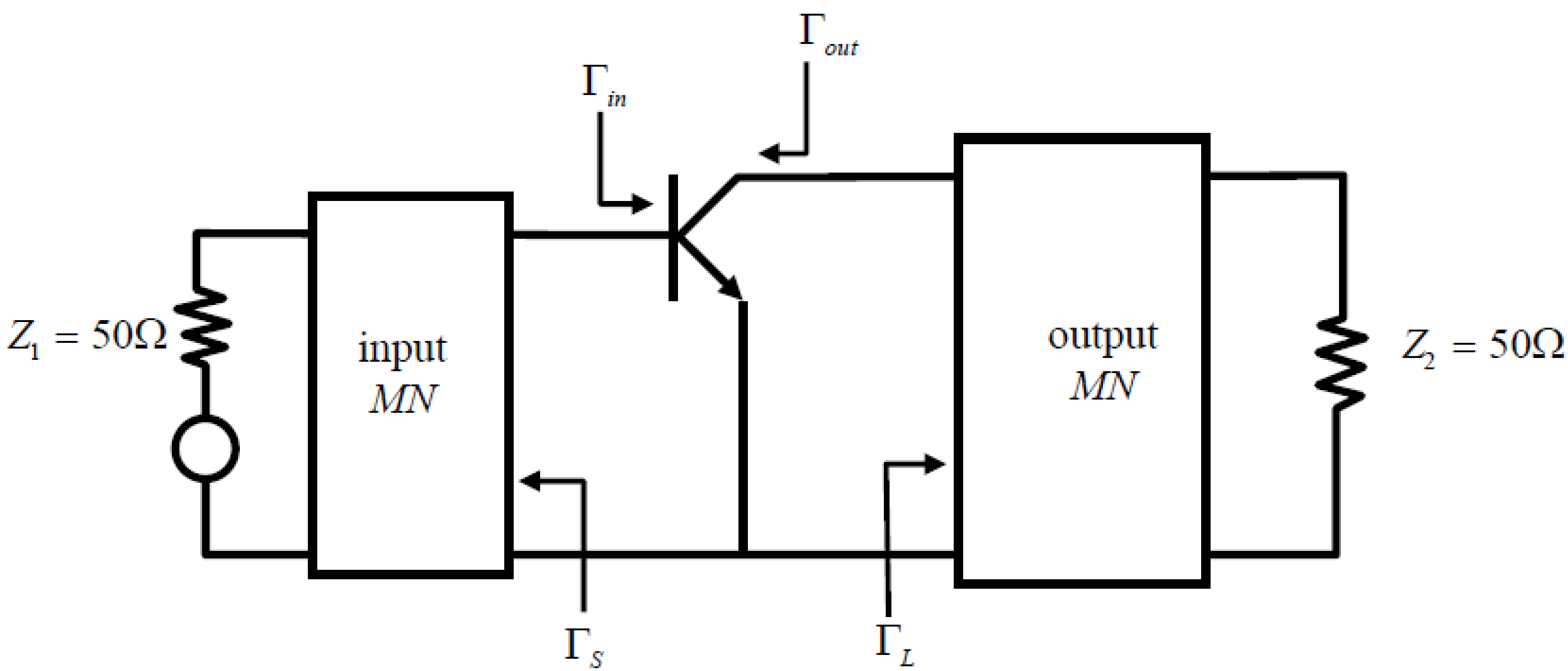
- balanced amplifier, distributed amplifier, differential amplifier

Power amplifier

- nonlinear operation

But, life is generally not that straightforward because $|S_{21}|^2$ is often much less than the optimum gain that you could obtain from a given transistor. You must add matching networks to transform Z_O to a more suitable Γ_S and Γ_L.

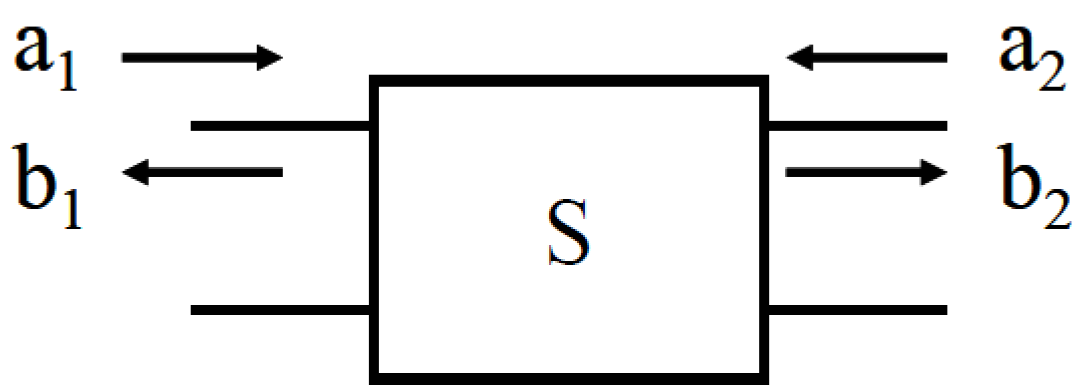
AMPLIFIER BLOCK DIAGRAM



How do we calculate gain from s-parameters?

Recall the definition of the S parameters:

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$



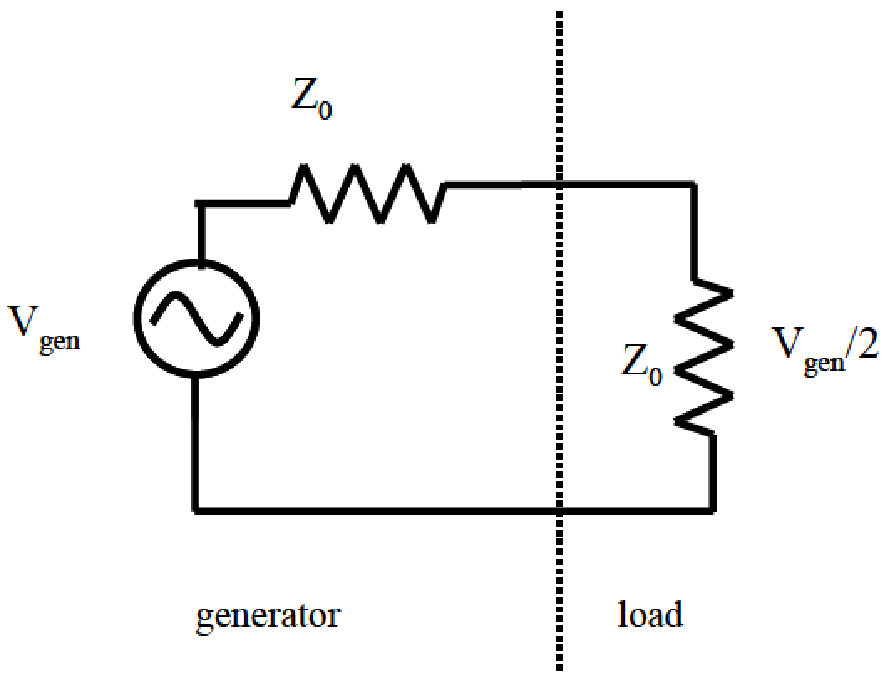
A transistor

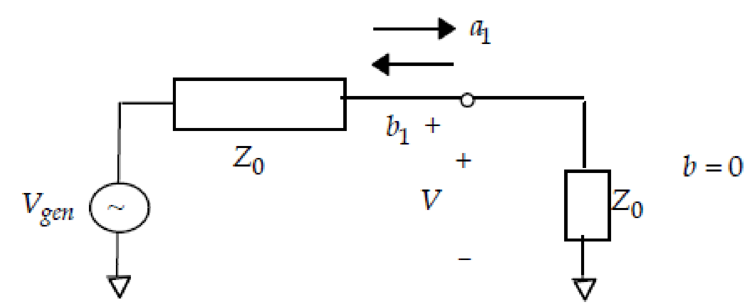
available power:

P_{AVS} = max power output from a source with impedance Z_S that can be absorbed into a load.

let $Z_S = Z_0,\ Z_L = Z_S^* = Z_0$ (in this case)

because maximum power transfer occurs when we have a conjugate match





$$a_1 = \frac{V^+}{\sqrt{Z_0}} \quad and\ b_1 = 0; \qquad V^+ = V_{gen}\left(\frac{Z_0}{Z_0 + Z_0}\right) = \frac{Vgen}{2} \quad and\ \ V^- = 0$$

$$\boxed{P_{load} = P_{AVS} = \frac{1}{2} a_1 a_1^* = \frac{V_{gen}^2}{8Z_0}}$$

- We see that the available power is independent of load impedance. Even if the load is not matched, available power remains constant. Actual power in the load is reduced however.

- Generator output power is calibrated and displayed as available power.

Actual Load Power

$$P_{\text{Load}} = \frac{1}{2}|a_1|^2 - \frac{1}{2}|b_1|^2 = \frac{1}{2}\text{Re}\left[I_1 V_1^*\right]$$

or

$$\boxed{P_{Load} = P_{AVS}(1 - |S_{11}|^2)}$$

Reflected Power $b_1 = a_1 S_{11}$

$$P_R = \frac{1}{2}|b_1|^2 = \frac{1}{2}|a_1|^2 |S_{11}|^2 = P_{AVS}|S_{11}|^2$$

$$|S_{11}|^2 = \frac{\text{Power reflected from input}}{\text{Power incident on input}} = \frac{|b_1|^2}{|a_1|^2}$$

$$|S_{22}|^2 = \frac{\text{Power reflected from network output}}{\text{Power incident on output}} = \frac{|b_2|^2}{|a_2|^2}$$

Consider the forward transmission and calculate the transducer power gain:

$$S_{21} = \frac{2V_{out}}{V_{gen}} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

In general, for an arbitrary R_S and R_L,

$$P_{AVS} = \frac{V_{gen}^2}{8R_S} \qquad\qquad P_L = \frac{V_{out}^2}{2R_L}$$

The definition of transducer power gain:

$$G_T = \frac{P_L}{P_{AVS}}$$

So, for the special case where $R_S = R_L = Z_O$,

$$|S_{21}|^2 = \frac{4V_{out}^2}{V_{gen}^2} = \frac{V_{out}^2}{2Z_O}\frac{8Z_O}{V_{gen}^2} = G_T$$

Reflected Power $b_1 = a_1 S_{11}$

$$P_R = \frac{1}{2}|b_1|^2 = \frac{1}{2}|a_1|^2 |S_{11}|^2 = P_{AVS}|S_{11}|^2$$

$$|S_{11}|^2 = \frac{\text{Power reflected from input}}{\text{Power incident on input}} = \frac{|b_1|^2}{|a_1|^2}$$

$$|S_{22}|^2 = \frac{\text{Power reflected from network output}}{\text{Power incident on output}} = \frac{|b_2|^2}{|a_2|^2}$$

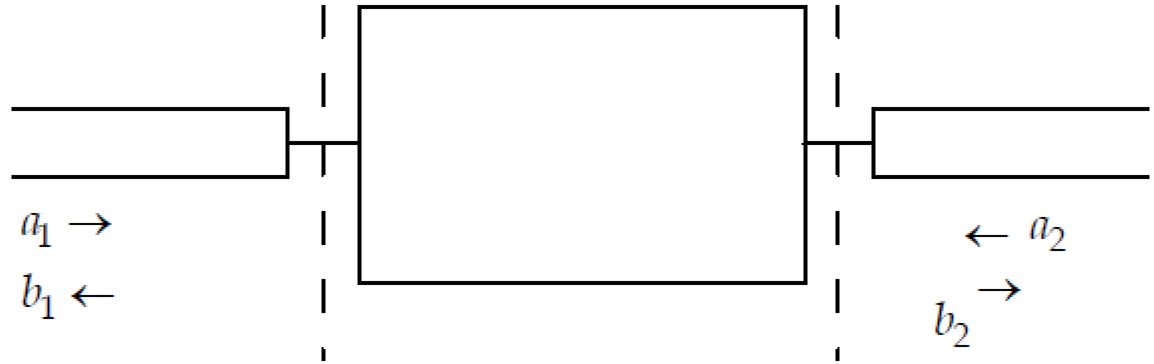
Similarly,

$$\frac{1}{2}|a_2|^2 = \text{Power incident on output}$$

$$= \text{Reflected power from load}$$

$$\frac{1}{2}|b_1|^2 = \text{Power reflected from input port}$$

$$\frac{1}{2}|b_2|^2 = \text{Power incident on load from the network}$$



Also, by definition, transducer gain $= \frac{P_{\text{load}}}{P_{\text{avs}}} = G_T$ even if

1. load isn't matched to network and
2. input of network not matched to generator

Here, $P_{Load} = |b_2|^2 (1 - |\Gamma_L|^2)$

S_{21} is defined in terms of transducer gain for the special case of where $Z_L = Z_0$:

$$|S_{21}|^2 = \left. \frac{|b_2|^2}{|a_1|^2} \right|_{a_2 = 0}$$

$|S_{21}|^2 =$ transducer gain with source and load Z_0

Similarly, $|S_{12}|^2 =$ reverse transducer power gain

$\frac{1}{2}|b_2|^2 =$ power incident on load (and is absorbed since Γ_L=0)

$\frac{1}{2}|a_1|^2 =$ source available power

Impedance Matching

Why do we impedance match? Power transfer is reduced when we have a mismatch.

Example: Suppose we have a 1V source with 100 ohms source resistance, Rs. The available power is the largest power that can be extracted from the source, and this is only possible when matched: $R_L = R_S$.

$$P_{avs} = \frac{V_{gen}^2}{8R_S} = 1.25\ mW$$

If we were to attach a 1000Ω load, $P_{Load} = \frac{1}{2}\mathrm{Re}\{V_L I_L^*\}$

$V_L = V_{gen}\ (1000/1100)$ $I_L = V_{gen}/1100$ $P_{LOAD} = 0.41\ mW$

Alternatively, we could calculate the reflection coefficient.

$$\Gamma_L = \frac{R_L/Z_0 - 1}{R_L/Z_0 + 1} = 0.818 \qquad P_L = P_{avs}\left(1 - |\Gamma_L|^2\right) = P_{avs}\,(0.33) = 0.41\,mW$$

So, if the source and load impedances are not matched, we can lose lots of power.

In this example, we have delivered only 33% of the available power to the load.

Therefore, if we want to deliver the available power into a load with a non-zero reflection coefficient, a matching network is necessary.

Power Gain Definition

- From the power components, 3 types of power gain can be defined.

$$\text{Power Gain } G_p = \frac{\text{Power delivered to load}}{\text{Input power to Amp.}} = \frac{P_L}{P_{in}} \qquad \textbf{(2.1a)}$$

$$\text{Available Power Gain } G_A = \frac{\text{Available load Power}}{\text{Available Input power}} = \frac{P_{Ao}}{P_{As}} \qquad \textbf{(2.1b)}$$

$$\text{Transducer Gain } G_T = \frac{\text{Power delivered to load}}{\text{Available Input power}} = \frac{P_L}{P_{As}} \qquad \textbf{(2.1c)}$$

The effective power gain

- G_P, G_A and G_T can be expressed as the S-parameters of the amplifier and the reflection coefficients of the source and load networks.

Kaynaklar

- https://www.ece.ucsb.edu/~long/ece145a/ampdesign.pdf
- Amplifiers, Prof. Tzong-Lin Wu. EMC Laboratory. Department of Electrical Engineering. National Taiwan University

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Sincerely,

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Kaynaklar

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